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Capacitor

SOLUTIONS

BEGINNER'S BOX-1

1. $\phi = \frac{q_{enc}}{\varepsilon_0} \because q_{enc} = 0 \therefore \phi = 0$

2. Zero, $\frac{1}{2}\frac{Q^2}{C}$

3. $U \propto Q^2$

$$\frac{U_1}{U_2} = \left(\frac{Q_1}{Q_2}\right)^2 \Rightarrow \frac{U_1}{U_2} = \left(\frac{1}{2}\right)^2$$

$\Rightarrow U_2 = 4W$

4. $R = n^{1/3}r$

$R = 8^{1/3} r$

$C' = 4\pi\in_0 R = 2(4\pi\in_0 r) = 2C$

5. $C = \frac{4\pi\in_0 b^2}{b-a}$

Here $C = 1\mu F$, $b - a = 1$ mm

$$\Rightarrow 1\times10^{-6} = \frac{1}{9\times10^9}\frac{b^2}{1\times10^{-3}}$$

On solving we get b = 3 m

6. $C = \frac{Q}{V} = \frac{2\times10^{16}\times1.6\times10^{-19}}{10}$

$= 3.2 \times 10^{-4}$ F

7. A Because $C = \frac{Q}{V} \propto \frac{1}{\text{slope}}$

$\Rightarrow C_A > C_B$

8. $Q = CV = 0.09$ C

$\Rightarrow U = \frac{Q^2}{2C} = 135$ J

9. According to question

$$\frac{1}{2}C_1V_1^2 = \frac{1}{2}C_2V_2^2$$

$$\frac{V_1}{V_2} = \sqrt{\frac{C_2}{C_1}}$$

BEGINNER'S BOX-2

1. Work = $U_F - U_i$

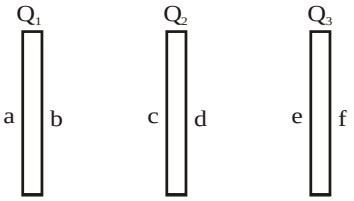
and charge remains constant

$\because C = \frac{\varepsilon_0 A}{d}$ $C' = \frac{\varepsilon_0 A}{2d}$

$V = V$ $V' = 2V$

$\therefore U_i = \frac{\varepsilon_0 AV^2}{2d}$ $U_F = \frac{\varepsilon_0 A(2V)^2}{2(2d)}$

2.



Changes on outer faces = $\frac{Q_1 + Q_2 + Q_3}{2}$

3. $C_{medium} = KC$

$\Rightarrow K = \frac{175\text{pF}}{50\text{pF}} = 3.5$

4. $C = \frac{\in_0\in_r A}{d}$; $U = \frac{1}{2}CV^2$

$= \frac{1}{2} \times (5.94 \times 10^{-11}) \times (12)^2 = 4.3 \times 10^{-9}$ J

5. $C = \frac{\in_0 A}{d - t + \frac{t}{K}}$ $\left(t = \frac{d}{2}, K = \infty\right)$

$$= \frac{\in_0 A}{\left(d - \frac{d}{2}\right) + \frac{d}{2K}} = \frac{2\in_0 A}{d} = 2C_0$$

6. $C' = KC_0 = 5C_0$

$U' = \frac{Q^2}{2C'} = \frac{W_0}{5}$

7. $\because$ Electric field $E = \frac{V}{d}$

$\therefore d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}$ m

Capacitance $C = \frac{\in_0\in_r A}{d}$

$\Rightarrow A = \frac{Cd}{\in_0\in_r} = \frac{88.5\times10^{-12}\times10^{-3}}{8.85\times10^{-12}\times10} = 10^{-3}$ m^2

8. (a) Charge on C $Q = CV = 10\times 20\ \mu C$

$= 200\ \mu C$

(b) Charge on C' $Q' = C'V = 10\times 4\times 20$

$= 800\ \mu C$

9. Since battery is connected so voltage across capacitor remains constant

Initial charge on capacitor

$Q = CV = 10 \times 10^{-6} \times 10 = 100\mu C$

When liquid of dielectric constant 5 is filled between plates then capacity becomes $C' = 5C = 50\mu F$

So final charge after placing liquid

$Q' = C'V = 50 \times 10^{-6} \times 10 = 500\mu C$

So additional charge flowing from battery to capacitor $\Delta Q = Q' - Q = 500 - 100 = 400\mu C$

10. Energy density $= \frac{1}{2}\epsilon_0 E^2$

$\because \quad E = \frac{V}{d}$

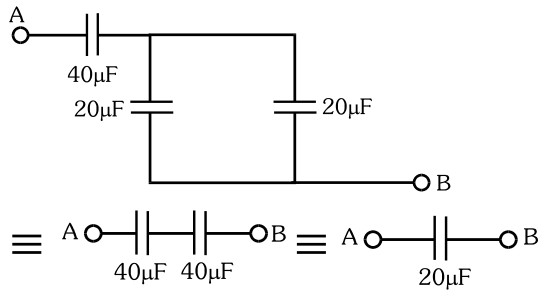
So energy density $= \frac{1}{2}\epsilon_0 \frac{V^2}{d^2}$

11. $\sigma_{big} = (n)^{\frac{1}{3}} \times \sigma_{small} = (64)^{\frac{1}{3}} \times \sigma_{small}$

$\sigma_{big} = 4 \times \sigma_{small}$

BEGINNER'S BOX-3

1. Given circuit can be reduced to



Therefore $C_{AB} = 20\ \mu F$

If $V_{AB} = 100$ V then charge flow through circuit

$= C_{AB}V_{AB} = (100)(20) = 2 \times 10^{-3}$ C

2. (a) Net capacitance $C_{net} = \frac{C}{3} = \frac{9}{3} = 3pF$

(b) in series $V = V_1 + V_2 + V_3 = 3V_1$

($\because V_1 = V_2 = V_3$ identical capacitor)

Potential difference across each $V_1 = \frac{V}{3} = \frac{120}{3} = 40V$

3. Common potential $V_{cm} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{2 \times 200 + 3 \times 400}{2+3} = 320$ V

Charge on $C_1 = Q_1 = C_1V_{cm} = 2 \times 320\ \mu C$

$= 640\ \mu C$

Charge on $C_2\ Q_2 = C_2V_{cm} = 3 \times 320\ \mu C$

$= 960\ \mu C$

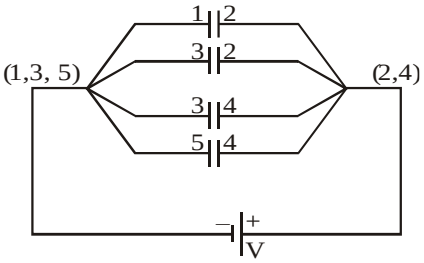
4. $C_{eff} = \frac{C_1C_2}{C_1 + C_2}$

$C_1 = \frac{K_1\epsilon_0 A}{d/2} = \frac{2K_1\epsilon_0 A}{d}$

$C_2 = 2K_2\frac{\epsilon_0 A}{d}$

$C_{eff} = \frac{2K_1K_2}{(K_1 + K_2)}\frac{\epsilon_0 A}{d}$

5.

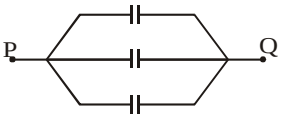


$Q = CV = \frac{\epsilon_0 A}{d}V$

Charge on plate $1 = -\frac{\epsilon_0 A}{d}V$

Charge on plate $4 = \frac{\epsilon_0 AV}{d} + \frac{\epsilon_0 A}{d}V = 2\frac{\epsilon_0 A}{d}V$

6.



$C_{PQ} = \frac{3\epsilon_0 A}{d}$

7. (a) Potential energy $U_{each} = \frac{1}{2}CV^2$

(b) Common potential $V_{cm} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{CV - CV}{C + C} = 0$

(c) Charge $Q'_1 = C_1V_{cm} = C(0) = 0$ &

$Q'_2 = C_2V_{cm} = C(0) = 0$

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(d) Potential energy

$$U'_1 = \frac{1}{2}C_1V^2_{cm} = 0 \ \& \ U'_2 = \frac{1}{2}C_1V^2_{cm} = 0$$

(e) Potential energy loss = $U_i - U_f$

$$= 2 \times \frac{1}{2}CV^2 - 0 = CV^2$$

BEGINNER'S BOX-4

1. (i) For capacitor

$$\frac{V_0}{R} = 10$$

$$\Rightarrow V_0 = 10R = 100 \text{ volt}$$

(ii) $i = 10e^{-t/RC}$

$$\Rightarrow 2.5 = 10e^{-2/RC}$$

$$\Rightarrow RC = \tau = \frac{1}{\ell n2} \ \& \ C = \frac{1}{10\ell n2}$$

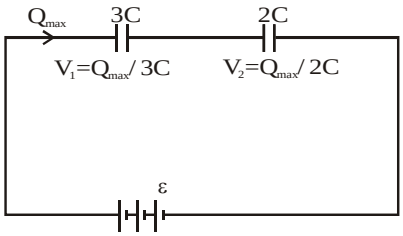
(iii) Total heat developed

= Total initial energy stored in capacitor.

$$= \frac{1}{2}CV^2 = \frac{500}{\ell n2}$$

(iv) Time - constant = $RC = \frac{1}{\ell n2}$

2.



$q_{1max} = 3C(1 \text{ kV})$; $q_{2max} = 2C(2 \text{ kV})$

$\Rightarrow Q_{max} = q_1$

So, $V_1 = 1$ kV ; $V_2 = 1.5$ kV

Safe voltages in each arm are mentioned.

$E_{safe} = 1+1.5 = 2.5$ kV

3. V decreases continuously from left to right except in conductor where it is constant.

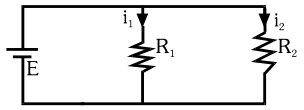
4. Total charge $Q = CV = 400 \times 100$

$= 40000 \ \mu C = .04$ C

$$\text{time taken} = \frac{\text{Total charge}}{\text{Charge rate}}$$

$$= \frac{0.04}{100\times10^{-6}} = 400 \text{ s}$$

5. (a)



At t = 0, capacitor has zero resistance, i.e., R_1 and R_2 are in parallel. The simple circuit is shown in fig.

$$i_1 = \frac{E}{R_1} \text{ and } i_2 = \frac{E}{R_2}$$

(b) At steady state ($t=\infty$), capacitor has infinite resistance. Hence, $i_1 = \frac{E}{R_1}$, $i_2 = 0$

(c) Final potential difference across capacitor is E. $\therefore$ Final energy stored U $= \frac{1}{2}CE^2$

(d) When switch is opened, capacitor will discharge through two resistance as R_1 and R_2 (both in series).

Hence, time constant $\tau_c = C(R_1 + R_2)$

(e) When switch is closed, capacitor will charged through resistance R_2. So $\tau = R_2C$

6. $H = \frac{1}{2}CV^2 = \frac{1}{2}\times2\times10^{-6}\times200\times200 = 0.04$ J